

# Quantization of Nambu-Goto Action in Four Dimensions

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# Dirac's statement

Quantization is replacing Poisson brackets with commutators.

It is Dirac who noticed that the statement is wrong sometimes.

See

P.A.M. Dirac, Yeshiva Univ. Press,  
1964

# What to do when the Poisson brackets and the constraints are inconsistent?

- a) Sustain the constraint equations, but the commutation relations will be compromised, at least, partially.
  - ex. Dirac brackets
- b) Sustain the commutation relations, but a part of the constraint equations will be given up.
  - ex. OCQ(old covariant quantization), QCD

My message: Try (a)!

# Naming of the Freedoms

$X^+, X^-$  : transverse

$X^S$  :  $p^\mu$  proportional

$X^L$  : orthogonal to  $X^+, X^-, X^S$

- By Lorentz transformation,  $X^S$  acts as a Lorentz scalar, and  $X^+, X^-$ , and  $X^L$  linearly mix as a three vector.

# Algebra

- Conventional string (OCQ)

$$[X^i(\sigma), \Pi^j(\sigma')] = -i\hbar\delta^{ij}\delta(\sigma' - \sigma)$$

$$L_m|*\rangle = 0 \quad (m > 0)$$

and

$$[X^S(\sigma), \Pi^S(\sigma')] = +i\hbar\delta(\sigma' - \sigma)$$

- Our method

$$[X^i(\sigma), \Pi^j(\sigma')] = -i\hbar\delta^{ij}\delta(\sigma' - \sigma)$$

$$L_m|*\rangle = 0 \quad (\forall m)$$

and

$$[X^S(\sigma), \Pi^S(\sigma')] = F(L'_m)$$

# No Other Choice

- # freedom by  $L_m = 0$

||

- # freedom of  $X^S$  &  $\Pi^S$

➔ No other choice than to change  $[X^S, \Pi^S]$ .  
And this is manifestly covariant because  $X^S$  is a Lorentz scalar.



# Consequence of our algebra:

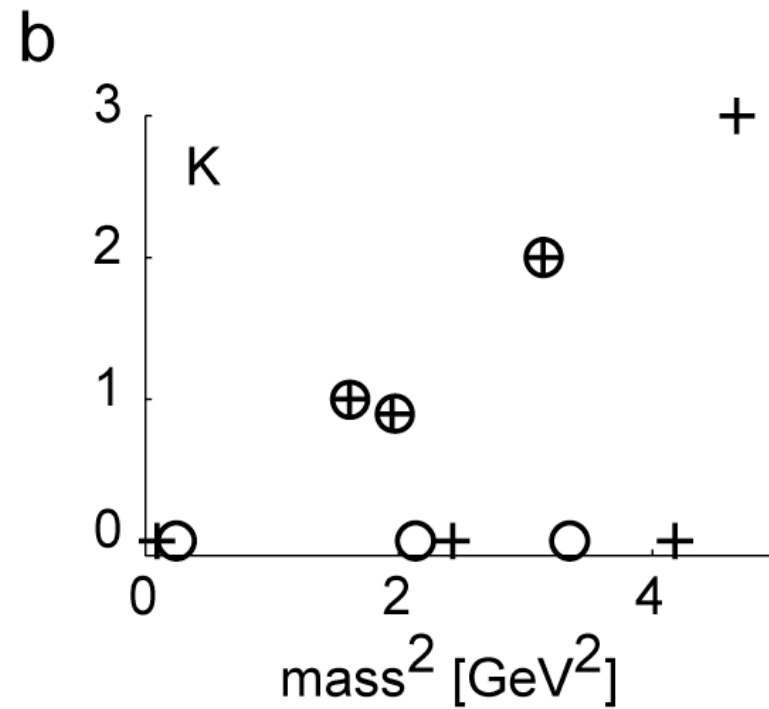
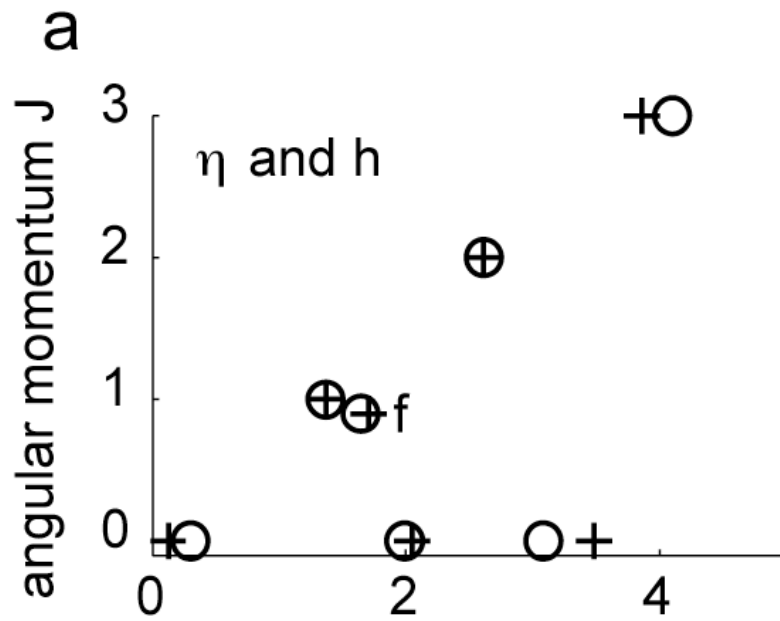
The possible states:

State	spin	$(L_0 - a) = 0$
$ 0\rangle$	0	$p^2 = -2a$
$\alpha_{-1}^i  0\rangle$	1	$p^2 = 2 - 2a$
$\sum_i \alpha_{-1}^i \alpha_{-1}^i  0\rangle$	0	$p^2 = 4 - 2a - (D - 1)/p^2$
$\alpha_{-2}^i  0\rangle$	1	$p^2 = 2 - 2a - 4/p^2$
$\alpha_{-1}^i \alpha_{-1}^j  0\rangle$	2	$p^2 = 4 - 2a$
$\alpha_{-3}^i  0\rangle$	1	
$\sum_i \alpha_{-2}^i \alpha_{-1}^i  0\rangle$	0	$p^2 = 4 - 2a - (D - 1)/p^2$
$\epsilon_{kij} \alpha_{-2}^i \alpha_{-1}^j  0\rangle$	1	$p^2 = 4 - 2a - (D - 1)/p^2$
$\alpha_{-2}^i \alpha_{-1}^j  0\rangle$	2	
$\alpha_{-1}^i (\alpha_{-1}^j \alpha_{-1}^j)  0\rangle$	1	
$\zeta_{ijk} \alpha_{-1}^i \alpha_{-1}^j \alpha_{-1}^k  0\rangle$	3	
...		

No negative norm states!

I intended it to be  
the fundamental string,  
surprisingly I found coincidence  
with:

# Real Meson Spectrum



# Why need not $D = 26$ ?

- Conventional covariant:

$(L_{-2} + (3/2)L_{-1}^2)|0\rangle$  : a negative norm state

Norm:  $(D - 26)/2$

- Ours (covariant):

$$L_{-1}|0\rangle = 0$$

$$L_{-2}|0\rangle = 0$$

Then, no negative norm state!

# Summary

- Don't give up the Constraints!
- A Consistent Algebra
- Agreement of Meson Spectra

This is merely a part of Revising  
Quantization of all the gauge  
field theories.

See also: [arXiv:0909.4135](https://arxiv.org/abs/0909.4135)

Same method to gauge field