

# An alternative proposal for string quantization

Tsuguo MOGAMI \*

RIKEN, Hirosawa 2-1, Wako-shi, Saitama, 351-0198 Japan

November 14, 2011

## Abstract

This article is written so as to make the reader easily follow the calculation to obtain the spectrum, which is the most persuading part. See arXiv:1005.2726v2 for the physical explanations and the full details.

We assume that the total momentum of the string is  $p_\mu = (E, 0, 0, 0)$  without any loss of generality owing to a Lorentz boost. Lorentz invariance will seemingly be broken because of this. The constraints are defined as:

$$L_m = \frac{1}{2} \sum_{n=-\infty}^{\infty} (\alpha_{m-n}^i \alpha_n^i - \alpha_{m-n}^0 \alpha_n^0) = 0, \quad (1)$$

where  $\alpha_n^i = \Pi_n^i - inX_n^i/2$  and  $\alpha_n^0 = \Pi_n^0 - inX_n^0/2$ . Roman indices  $i$  and  $j$  are spatial indices. Note that  $\alpha_0^\mu = \Pi_0^\mu$  is the operator of the total momentum, and we will work in the subspace with the eigenvalue  $\Pi_0^\mu = (E, 0, 0, 0)$ .

## The assumptions of the Conventional string quantization (OCQ)

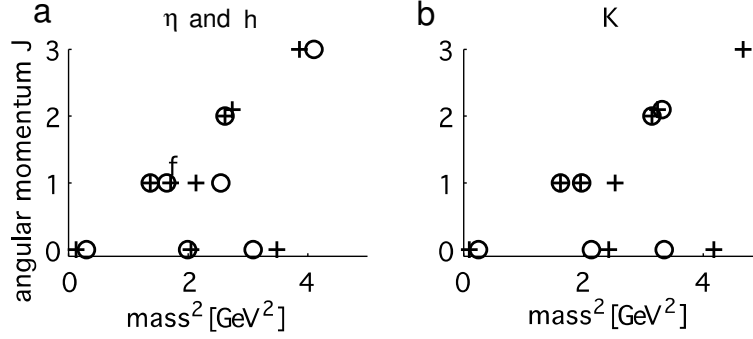
$$\begin{aligned} [\alpha_m^0, \alpha_{-n}^0] &= -n\delta_{mn} \\ [\alpha_m^i, \alpha_{-n}^j] &= n\delta^{ij}\delta_{mn} \\ L_m|*\rangle &= 0 \quad (m > 0) \end{aligned}$$

## The assumptions of Our string quantization

$$\begin{aligned} [\alpha_m^0, \alpha_{-n}^0] &: \text{no assumption} \\ [\alpha_m^i, \alpha_{-n}^j] &= \hbar\delta^{ij}\delta_{mn} \\ L_m|*\rangle &= 0 \quad (\forall m) \end{aligned}$$

---

\*e-mail: mogami@brain.riken.go.jp



**The consequences: the states and masses from our assumptions**

State	spin	mass <sup>2</sup>
$ 0\rangle$	0	$E^2 = -2a$
$\alpha_{-1}^i 0\rangle$	1	$E^2 = 2 - 2a$
$\sum_i \alpha_{-1}^i \alpha_{-1}^i  0\rangle$	0	$E^2 = 4 - 2a - (D - 1)/E^2$
$\alpha_{-2}^i 0\rangle$	1	$E^2 = 2 - 2a - 4/E^2$
$\alpha_{-1}^i \alpha_{-1}^j  0\rangle$	2	$E^2 = 4 - 2a$
$\alpha_{-3}^i 0\rangle$	1	
$\sum_i \alpha_{-2}^i \alpha_{-1}^i  0\rangle$	0	$E^2 = 4 - 2a - (D - 1)/E^2$
$\epsilon_{kij} \alpha_{-2}^i \alpha_{-1}^j  0\rangle$	1	$E^2 = 4 - 2a - (D - 1)/E^2$
$\alpha_{-2}^i \alpha_{-1}^j  0\rangle$	2	
$\alpha_{-1}^i (\alpha_{-1}^j \alpha_{-1}^j)  0\rangle$	1	
$\zeta_{ijk} \alpha_{-1}^i \alpha_{-1}^j \alpha_{-1}^k  0\rangle$	3	
...		

Numerical solution were able to be obtained for the particles with the empty mass cells.  $D$  is the number of the spacetime dimensions.

The result showed agreement with the real mesons (the figure). Further notable agreements were discussed in the text. In the figure, the experimental masses (circles) of light mesons and our theoretical predictions (crosses) are plotted for a)  $\eta$  and  $h$  mesons ( $I = 0$ ), and b) Kaons ( $I = 1/2$ ). Two Kaon states are predicted here. Pions showed agreements as well (the text). The meson masses are taken from “review of particle physics” (Particle Data Group, Phys. Lett. B667, 1 (2008)).

## The calculation for the result above

We adopt a metric with signature  $\{-+++ \dots\}$ , repeated indices are always summed, and  $|*\rangle$  designates “any physical state”.

We may use operator equations  $L_m = 0$  in the physical space. To sustain them, we may solve this equation for  $\alpha_n^0$  perturbatively up to any order in  $1/E$  by repeatedly using

$$L_n = L'_n - E\alpha_n^0 - \frac{1}{2} \sum_{m=-\infty; m \neq 0, -n}^{\infty} \alpha_{-m}^0 \alpha_{n+m}^0 = 0, \quad (2)$$

where  $L'_m \equiv \frac{1}{2} \sum_i \sum_{n=-\infty; n \neq 0, m}^{\infty} \alpha_{m-n}^i \alpha_n^i$ . Then we obtain

$$\alpha_n^0 = \frac{1}{E} L'_n - \frac{1}{2E^3} \sum_{k \neq 0, n} L'_{n-k} L'_k + \frac{1}{E^5} O(L'^3) - \dots. \quad (3)$$

Putting this equation into  $L_0$ , we have

$$\begin{aligned} L_0 &= -\frac{E^2}{2} + L'_0 - \frac{1}{2E^2} \sum_{k \neq 0} L'_{-k} L'_k \\ &+ \frac{1}{4E^4} \left( \sum_{k \neq 0; l \neq 0, -k} L'_{-k-l} L'_l L'_k + \sum_{k \neq 0; l \neq 0, k} L'_{-k} L'_{k-l} L'_l \right) - \dots, \quad (4) \end{aligned}$$

which being zero will determine the energy  $E$ . Please note that neither ordering change nor commutation relation of  $L'_k$ 's was used up to here. Therefore these relations are valid quantum mechanically.

The ground state was defined as

$$\begin{aligned} \alpha_n^i |0\rangle &= 0, \\ \Pi_0^\mu |0\rangle &= p^\mu |0\rangle. \end{aligned} \quad (5)$$

Then from eq.(3), we have

$$\alpha_n^0 |0\rangle = 0 \quad (6)$$

for any positive  $n$ .

Now let us list up the states and their masses. We normal order the  $L'_k$ 's in  $L_0$ . This change of ordering gives extra terms, which have small factors such as  $\zeta(-1) = -1/12$ ,  $\zeta(-2) = 0$ , and so on. We will omit these terms in this article since they only give small corrections to the masses.

The first state is obviously the vacuum

$$|0\rangle. \quad (7)$$

It represents a spin-0 state, and its invariant mass is  $E^2 = -a$  because of the constraint  $(L_0 - a)|0\rangle = 0$ . Here a negative constant  $a$  was introduced to absorb the ordering ambiguity of  $L_0$ .

In the next level, the physical states are

$$\alpha_{-1}^i|0\rangle, \quad (8)$$

which form a spin-1 representation. We have

$$\alpha_{-1}^0|0\rangle = 0 \quad (9)$$

applying (3) with  $n = -1$ . This existence of 3 polarizations and absence of 0-direction polarization agrees with the classical Nambu-Goto string. The invariant mass of the level-1 state is  $E^2 = 2 - 2a$  because

$$(L_0 - a)\alpha_{-1}^i|0\rangle = (-E^2/2 + 1 - a)\alpha_{-1}^i|0\rangle = 0 \quad (10)$$

using  $L'_1\alpha_{-1}^i|0\rangle = 0$ .

In the second level, the physical states are

$$\alpha_{-1}^i\alpha_{-1}^i|0\rangle, \quad \alpha_{-2}^i|0\rangle, \quad \zeta_{ij}\alpha_{-1}^i\alpha_{-1}^j|0\rangle, \quad (11)$$

where  $\text{tr}\zeta_{ij} = 0$  and each state has spin 0, spin 1, and spin 2 respectively. The relations

$$\alpha_{-2}^0|0\rangle = \frac{1}{2}\alpha_{-1}^i\alpha_{-1}^i|0\rangle/E, \quad (12)$$

$$\alpha_{-1}^0\alpha_{-1}^i|0\rangle = \alpha_{-2}^i|0\rangle/E, \quad (13)$$

which comes from eq.(3), make it possible to represent the other level-2 states:  $\alpha_{-2}^0|0\rangle$ ,  $\alpha_{-1}^0\alpha_{-1}^0|0\rangle$ ,  $\alpha_{-1}^0\alpha_{-1}^i|0\rangle$ , and  $\alpha_{-1}^i\alpha_{-1}^0|0\rangle$  as linear combinations of the above physical states (11) or zero. In this way, any product of  $\alpha_n^i$ 's and  $\alpha_n^0$ 's may be reduced to a product of only  $\alpha_n^i$ 's.

The equation  $(L_0 - a)|*\rangle = 0$  gives  $E^2/2 = 2 - a$  for the spin-2 state and

$$E^2 = 4 - 2a - (D - 1)/E^2 \quad (14)$$

for the spin-0 state, whose solution is  $E^2 \simeq 3$  for small  $|a|$ . For the spin-1 state in this level, we have

$$E^2 = 4 - 2a - 4/E^2, \quad (15)$$

and its solution is  $E^2 \simeq 2$  for small  $|a|$ , which means this level-2 state have a mass close to that of the level-1 state, and nonzero  $a$  will push the mass upward. We did not take the other solutions of these quadratic equations. (An argument is given in the text.)

The spectrum of level-3 states may be calculated in the same way. The level-3 calculation required numerically solving an eigenvalue problem. The higher level spectra may be obtained further in the same way.